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REPORT NO. 710

THE CHI SQUARE TEST OF GOODNESS OF FIT: EXACT CRITICAL VALUES FOR THE CASE OF EQUIPROBABLE ALTERNATIVES

Progress Report
by

James N. Cronholm, M. S.

5 December 1966

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Acknowledgments

The computer program for the calculations on which the tables in this report are based was written by John C. Hoff. Carl E. Guthrie helped compile Table 1.

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THE CHI SQUARE TEST OF GOODNESS OF FIT: EXACT CRITICAL VALUES FOR THE CASE OF EQUIPROBABLE ALTERNATIVES

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Experimental Psychology Division
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ABSTRACT

THE CHI SQUARE TEST OF GOODNESS OF FIT: EXACT CRITICAL VALUES FOR THE CASE OF EQUIPROBABLE ALTERNATIVES

OBJECTIVE

To present a table of exact critical values of the chi square criterion of goodness of fit for the case of equiprobable alternatives, and to compare the exact test with the more familiar continuous approximation.

RESULTS

Exact critical values are given for the chi square criterion for the equiprobable case when there are n observations and k categories ($2 \leq n \leq 15$, $2 \leq k \leq 15$). Nominal significance levels given are: $\alpha \leq .200$, .100, .050, .025, .010, .005, .001. Exact significance levels are presented for $\alpha \leq .050$, and it is shown that the true significance level of the approximation with $\alpha = .050$ exceeds .050 in 27 percent of the cases examined.

CONCLUSIONS

Statistical tests of the hypothesis of equal category probabilities are conservative with respect to Type 1 errors if the proposed tables are used. The standard chi square test of this hypothesis is not uniformly conservative.

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THE CHI SQUARE TEST OF GOODNESS OF FIT: EXACT CRITICAL VALUES FOR THE CASE OF EQUIPROBABLE ALTERNATIVES

When n independent observations are distributed over k mutually exclusive and exhaustive categories, the chi square criterion, given by

$$(1) \qquad \chi^2 = \sum_{i=1}^{k} \frac{(n_i - np_i)^2}{np_i},$$

is frequently used as a measure of the goodness of fit of observed frequencies, n_i, to expected frequencies, np_i. In the special case considered here, the population category probabilities are assumed to be equal, $p_i = 1/k$ $(i = 1, 2, \ldots, k)$. The purpose of this paper is to present a table of exact critical values of χ^2 and compare the exact test with its more familiar continuous approximation.

Table 1 gives exact critical values of χ^2 for $2 \leq n \leq 15$, $2 \leq k \leq 15$, and selected significance levels, $\alpha \leq .200, .100, .050, .025, .010, .005, .001$. The entries in Table 1 were extracted from recently prepared tables of exact cumulative sampling probabilities produced by computer expansion of a generating function [3]. While it is possible to obtain the required sampling distributions by hand, the computations are very lengthy even when n and k are relatively small: the 196 tables of cumulative sampling probabilities from which Table 1 was derived required approximately 15 minutes of IBM 7094 computing time.

Some parts of Table 1 can be obtained from earlier work. Cartwright [1] tabulated critical values of his index of agreement, Alpha, for $3 \leq n \leq 5$, $4 \leq k \leq 16$. It can easily be shown that Alpha = $(\chi^2 + n - k)/k(n - 1)$ when the $p_i = 1/k$ $(i = 1, 2, \ldots, k)$. However, Cartwright's critical values were obtained from interpolation between points on the uncumulated distributions. Cronholm [2, p. 27] tabulated uncumulated sampling probabilities of Alpha for $2 \leq n \leq 13$, $2 \leq k \leq 11$. But these probabilities are only given to four places beyond the decimal point and their cumulation is probably inaccurate [2, p. 31]. Various authors have also calculated exact sampling distributions of χ^2 for particular values of n and k. For example, Walker and Lev [6] consider the two cases k = 3, n = 6, 12 in their introduction to the continuous approximation. Other examples include [4, 5, 7, 8].

To test an hypothesis of equal category probabilities, Table 1 is entered at α, n, and k. If the observed value of the criterion (1) is

equal to or greater than the value tabulated, the hypothesis may be rejected at or beyond the level specified by α. Because the exact sampling distribution of χ^2 is discrete and rather irregular with respect to both χ^2 and its sampling probabilities, Table 1 gives the smallest obtainable value of χ^2 with a cumulative probability of $1 - \alpha$ or more.

As an example, suppose that n = 5 observations are distributed over k = 10 categories and the value of χ^2 is found to be 29.0000. To test the hypothesis that the sample came from a population with equal category probabilities, $p_i = 1/10$ ($i = 1, 2, \ldots, 10$), enter Table 1 at n = 5, k = 10, and $\alpha = .05$, say. The critical value is found to be 21.0000, and since this is less than the observed value of 29.0000, the hypothesis may be rejected. The observed value is also significant at the .005 level. Notice that the exact test does not require that expected frequencies, n/k, be greater than some minimum - usually 5. In the present example they are equal to .5. Thus, the problems of pooling categories or discarding data do not arise.

Table 2 gives the exact significance levels of the corresponding entries in Table 1 at the nominal .050 level. All of the actual significance levels are less than .050, but some are substantially less, e.g., n = 4, k = 8. The average exact value of α is .0316, so that the exact test is somewhat conservative.

Table 3 has been prepared in order to make certain comparisons between the exact test of Table 1 and the standard χ^2 distribution. The entries in Table 3 are the exact significance levels of the smallest observable values of χ^2 which yield significance at the .050 level using the continuous approximation. In other words, these entries are the true significance levels operating when the standard test is used with $\alpha = .050$. If all the entries in Table 3 were equal to the corresponding entries in Table 2, the two tests would be equivalent with respect to Type 1 errors over the range of n and k covered.

A comparison of Tables 2 and 3 will show that this is not generally the case. Two entries in Table 3 (n = 12, k = 7; n = 15, k = 5) are less than the corresponding entries in Table 2, 141 are equal (including the blanks where no obtainable value of χ^2 yields significance), and 53 (underlined) entries in Table 3 exceed .050. While the average true significance level for the approximate test is .0431, in 53 cases significance will be claimed at the .050 level when the true value of α is greater than .050. Unfortunately, these 53 cases are scattered rather uniformly over the table so that no simple rule involving n and k can insure a conservative test using the approximation. However,

the most serious errors occur when n is small, e.g., $n = 2$, $k = 9$, 10, 11, and $n = 3$, $k = 3$: as a compromise, if $n > 6$, the true significance levels will be no greater than .0704 ($n = 12$, $k = 3$) when the approximate test is used with $\alpha = .050$. This will be true regardless of the size of the expected frequencies, provided they are equal and $n, k \leq 15$. A better rule is to use Table 1.

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TABLE 1

Critical values of χ^2 for $2 \le k \le 15$, $2 \le n \le 15$, and $p_i = 1/k$

$\alpha = .200$

n\k	2	3	4	5	6	7	8
2	-	-	-	8.0000	10.0000	12.0000	14.0000
3	-	6.0000	9.0000	12.0000	15.0000	18.0000	21.0000
4	4.0000	8.0000	12.0000	8.5000	8.0000	10.0000	12.0000
5	5.0000	5.2000	5.4000	8.0000	10.6000	10.4000	12.6000
6	6.0000	4.0000	6.0000	9.0000	12.0000	10.3333	12.6667
7	3.5714	4.5714	7.2857	8.0000	7.5714	10.0000	12.4286
8	4.5000	4.0000	6.0000	7.0000	8.5000	9.5000	12.0000
9	2.7778	4.6667	5.6667	7.1111	7.6667	10.4444	11.4444
10	3.6000	3.8000	5.2000	7.0000	8.0000	9.6000	10.8000
11	4.4545	4.5455	5.3636	6.7273	8.0909	10.0000	11.5455
12	3.0000	4.5000	5.3333	6.3333	8.0000	9.0000	10.6667
13	3.7692	3.8462	5.1538	6.6154	7.7692	9.0769	11.0000
14	2.5714	4.0000	5.4286	6.7143	8.2857	9.0000	11.1429
15	3.2667	3.6000	5.0000	6.6667	7.8000	9.7333	11.1333

n\k	9	10	11	12	13	14	15
2	16.0000	18.0000	20.0000	22.0000	24.0000	26.0000	28.0000
3	24.0000	27.0000	30.0000	33.0000	36.0000	39.0000	22.0000
4	14.0000	16.0000	18.0000	20.0000	22.0000	24.0000	26.0000
5	14.8000	13.0000	14.8000	16.6000	18.4000	20.2000	22.0000
6	15.0000	14.0000	16.0000	18.0000	20.0000	22.0000	24.0000
7	12.2857	14.4286	16.5714	18.7143	17.1429	19.0000	20.8571
8	12.2500	14.5000	16.7500	16.0000	18.0000	20.0000	22.0000
9	12.0000	14.3333	14.2222	16.3333	18.4444	20.5556	22.6667
10	13.4000	14.0000	14.2000	16.4000	18.6000	18.0000	20.0000
11	12.7273	13.5455	16.0000	16.2727	18.5455	18.2727	20.3636
12	12.0000	13.0000	15.5000	16.0000	18.3333	18.3333	20.5000
13	12.6154	13.9231	14.9231	15.6154	18.0000	18.2308	20.4615
14	11.7143	13.1429	14.2857	15.1429	17.5714	18.0000	20.2857
15	12.0000	13.6667	15.0667	16.2000	17.0667	17.6667	20.0000

TABLE 1 (Continued)

Critical values of χ^2 for $2 \leq k \leq 15$, $2 \leq n \leq 15$, and $p_i = 1/k$

$\alpha = .100$

n\k	2	3	4	5	6	7	8
2	-	-	-	-	-	-	-
3	-	-	9.0000	12.0000	15.0000	18.0000	21.0000
4	-	8.0000	12.0000	16.0000	11.0000	13.5000	12.0000
5	5.0000	10.0000	8.6000	8.0000	10.6000	13.2000	15.8000
6	6.0000	7.0000	7.3333	9.0000	12.0000	15.0000	18.0000
7	7.0000	5.4286	7.2857	10.8571	11.0000	12.0000	12.4286
8	4.5000	6.2500	7.0000	8.2500	10.0000	13.0000	14.0000
9	5.4444	6.0000	6.5556	8.2222	10.3333	12.0000	13.2222
10	6.4000	5.0000	7.6000	8.0000	10.4000	12.4000	12.4000
11	4.4545	5.0909	6.8182	7.6364	10.2727	11.2727	13.0000
12	5.3333	6.0000	6.6667	8.0000	10.0000	11.3333	13.3333
13	3.7692	5.6923	7.0000	8.1538	9.6154	11.2308	12.2308
14	4.5714	5.2857	7.1429	8.1429	10.0000	11.0000	12.2857
15	5.4000	5.2000	6.6000	8.0000	9.4000	11.6000	13.2667

n\k	9	10	11	12	13	14	15
2	-	18.0000	20.0000	22.0000	24.0000	26.0000	28.0000
3	24.0000	27.0000	30.0000	33.0000	36.0000	39.0000	42.0000
4	14.0000	16.0000	18.0000	20.0000	22.0000	24.0000	26.0000
5	18.4000	17.0000	19.2000	21.4000	23.6000	25.8000	22.0000
6	15.0000	17.3333	19.6667	22.0000	24.3333	22.0000	24.0000
7	14.8571	17.2857	19.7143	18.7143	20.8571	23.0000	25.1429
8	14.5000	17.0000	16.7500	19.0000	21.2500	23.5000	25.7500
9	16.0000	16.5556	16.6667	19.0000	21.3333	20.5556	22.6667
10	15.2000	16.0000	18.6000	18.8000	21.2000	20.8000	23.0000
11	14.3636	15.3636	18.0000	18.4545	20.9091	23.3636	23.0909
12	15.0000	16.3333	17.3333	18.0000	20.5000	20.6667	23.0000
13	14.0000	15.4615	16.6154	19.3077	20.0000	20.3846	22.7692
14	14.2857	16.0000	17.4286	18.5714	19.4286	22.0000	22.4286
15	14.4000	15.0000	16.5333	17.8000	20.5333	21.4000	22.0000

TABLE 1 (Continued)

Critical values of χ^2 for $2 \le k \le 15$, $2 \le n \le 15$, and $p_i = 1/k$

$\alpha = .050$

n\k	2	3	4	5	6	7	8
2	-	-	-	-	-	-	-
3	-	-	-	12.0000	15.0000	18.0000	21.0000
4	-	8.0000	12.0000	16.0000	20.0000	24.0000	28.0000
5	-	10.0000	15.0000	12.0000	15.4000	13.2000	15.8000
6	6.0000	12.0000	11.3333	10.6667	14.0000	15.0000	18.0000
7	7.0000	8.8571	9.5714	10.8571	14.4286	14.0000	17.0000
8	8.0000	7.0000	8.0000	10.7500	11.5000	13.0000	16.0000
9	5.4444	8.0000	8.3333	10.4444	11.6667	13.5556	15.0000
10	6.4000	7.4000	8.4000	10.0000	11.6000	13.8000	15.6000
11	7.3636	6.7273	8.2727	10.3636	11.3636	13.8182	15.9091
12	5.3333	6.5000	8.0000	9.6667	12.0000	12.5000	14.6667
13	6.2308	6.6154	8.2308	9.6923	11.4615	13.3846	14.6923
14	7.1429	7.0000	8.2857	10.2857	11.7143	13.0000	14.5714
15	5.4000	6.4000	8.2000	9.3333	11.8000	13.4667	14.3333

n\k	9	10	11	12	13	14	15
2	-	-	-	-	-	-	-
3	24.0000	27.0000	30.0000	33.0000	36.0000	39.0000	42.0000
4	18.5000	21.0000	23.5000	20.0000	22.0000	24.0000	26.0000
5	18.4000	21.0000	23.6000	26.2000	28.8000	25.8000	28.0000
6	21.0000	24.0000	19.6667	22.0000	24.3333	26.6667	29.0000
7	17.4286	17.2857	19.7143	22.1429	24.5714	27.0000	29.4286
8	16.7500	19.5000	22.2500	22.0000	21.2500	23.5000	25.7500
9	16.0000	18.7778	21.5556	21.6667	24.2222	23.6667	26.0000
10	17.0000	18.0000	18.6000	21.2000	23.8000	26.4000	26.0000
11	16.0000	19.0000	20.0000	20.6364	23.2727	23.3636	25.8182
12	16.5000	18.0000	19.1667	22.0000	22.6667	23.0000	25.5000
13	16.7692	18.5385	20.0000	21.1538	22.0000	24.6923	25.0769
14	16.8571	17.4286	19.0000	22.0000	23.1429	24.0000	24.5714
15	16.8000	17.6667	19.4667	21.0000	22.2667	23.2667	26.0000

TABLE 1 (Continued)

Critical values of χ^2 for $2 \le k \le 15$, $2 \le n \le 15$, and $p_i = 1/k$

$\alpha = .025$

n\k	2	3	4	5	6	7	8
2	-	-	-	-	-	-	
3	-	-	-	-	-	18.0000	21.0000
4	-	-	12.0000	16.0000	20.0000	24.0000	28.0000
5	-	10.0000	15.0000	20.0000	15.4000	18.8000	22.2000
6	-	12.0000	11.3333	15.6667	14.0000	17.3333	20.6667
7	7.0000	8.8571	9.5714	12.2857	14.4286	18.0000	17.0000
8	8.0000	10.7500	11.0000	12.0000	14.5000	16.5000	16.0000
9	9.0000	8.0000	11.0000	12.6667	13.0000	16.6667	16.7778
10	6.4000	7.4000	10.0000	12.0000	12.8000	15.2000	17.2000
11	7.3636	8.9091	9.7273	11.2727	13.5455	15.0909	17.3636
12	8.3333	8.0000	10.0000	11.3333	13.0000	16.0000	16.0000
13	6.2308	8.0000	10.0769	12.0000	13.3077	15.5385	17.1538
14	7.1429	7.4286	9.4286	12.4286	13.4286	15.0000	16.8571
15	8.0667	8.4000	9.2667	12.0000	13.4000	15.3333	16.4667

n\k	9	10	11	12	13	14	15
2	-	-	-	-	-	-	-
3	24.0000	27.0000	30.0000	33.0000	36.0000	39.0000	42.0000
4	32.0000	36.0000	40.0000	44.0000	28.5000	31.0000	33.5000
5	18.4000	21.0000	23.6000	26.2000	28.8000	31.4000	34.0000
6	21.0000	24.0000	27.0000	30.0000	33.0000	26.6667	29.0000
7	20.0000	23.0000	22.8571	22.1429	24.5714	27.0000	29.4286
8	19.0000	22.0000	22.2500	25.0000	27.7500	30.5000	29.5000
9	20.0000	21.0000	21.5556	24.3333	27.1111	26.7778	29.3333
10	18.8000	20.0000	23.0000	23.6000	23.8000	26.4000	29.0000
11	19.2727	20.8182	22.0000	25.0000	25.6364	25.9091	28.5455
12	18.0000	19.6667	22.8333	24.0000	24.8333	27.6667	28.0000
13	18.1538	20.0769	21.6923	23.0000	26.0000	26.8462	27.3846
14	18.1429	20.2857	22.1429	23.7143	25.0000	26.0000	28.8571
15	18.0000	20.3333	22.4000	22.6000	24.0000	27.0000	28.0000

TABLE 1 (Continued)

Critical values of χ^2 for $2 \le k \le 15$, $2 \le n \le 15$, and $p_i = 1/k$

$\alpha = .010$

n\k	2	3	4	5	6	7	8
2	-	-	-	-	-	-	-
3	-	-	-	-	-	-	-
4	-	-	-	16.0000	20.0000	24.0000	28.0000
5	-	-	15.0000	20.0000	25.0000	30.0000	22.2000
6	-	12.0000	18.0000	15.6667	20.0000	17.3333	20.6667
7	-	14.0000	14.1429	13.7143	17.8571	18.0000	21.5714
8	8.0000	10.7500	12.0000	13.2500	16.0000	18.2500	20.0000
9	9.0000	12.6667	11.0000	16.0000	17.0000	16.6667	20.3333
10	10.0000	10.4000	13.2000	15.0000	16.4000	18.0000	18.8000
11	11.0000	8.9091	11.1818	14.0000	15.7273	17.6364	20.2727
12	8.3333	10.5000	12.0000	13.8333	16.0000	17.1667	20.0000
13	9.3077	9.3846	11.9231	13.5385	16.0769	17.6923	19.6154
14	10.2857	9.5714	12.2857	13.8571	15.1429	18.0000	19.1429
15	8.0667	10.0000	11.4000	13.3333	15.8000	17.2000	19.6667

n\k	9	10	11	12	13	14	15
2	-	-	-	-	-	-	-
3	-	27.0000	30.0000	33.0000	36.0000	39.0000	42.0000
4	32.0000	36.0000	40.0000	44.0000	48.0000	52.0000	56.0000
5	25.6000	29.0000	32.4000	26.2000	28.8000	31.4000	34.0000
6	24.0000	27.3333	30.6667	30.0000	33.0000	36.0000	39.0000
7	25.1429	23.0000	26.0000	29.0000	32.0000	35.0000	33.7143
8	21.2500	22.0000	25.0000	28.0000	31.0000	30.5000	33.2500
9	22.0000	23.2222	26.4444	27.0000	27.1111	29.8889	32.6667
10	22.4000	24.0000	25.2000	28.4000	29.0000	29.2000	32.0000
11	20.9091	22.6364	26.0000	27.1818	28.0000	31.0000	31.2727
12	22.5000	23.0000	24.6667	26.0000	29.1667	30.0000	33.0000
13	20.9231	23.1538	25.0769	26.6923	28.0000	31.1538	32.0000
14	20.7143	23.1429	25.2857	27.1429	28.7143	30.0000	31.0000
15	21.6000	23.0000	25.3333	25.8000	29.2000	30.7333	32.0000

TABLE 1 (Continued)

Critical values of χ^2 for $2 \le k \le 15$, $2 \le n \le 15$, and $p_i = 1/k$

$\alpha = .005$

n\k	2	3	4	5	6	7	8
2	-	-	-	-	-	-	-
3	-	-	-	-	-	-	-
4	-	-	-	-	20.0000	24.0000	28.0000
5	-	-	15.0000	20.0000	25.0000	30.0000	35.0000
6	-	12.0000	18.0000	24.0000	20.0000	24.3333	20.6667
7	-	14.0000	21.0000	19.4286	17.8571	22.0000	23.8571
8	-	16.0000	17.0000	17.0000	16.0000	20.0000	22.0000
9	9.0000	12.6667	14.5556	16.0000	18.3333	21.3333	20.3333
10	10.0000	14.6000	13.2000	15.0000	16.4000	20.8000	22.0000
11	11.0000	11.6364	13.3636	14.9091	16.8182	18.9091	21.7273
12	12.0000	10.5000	14.6667	15.5000	18.0000	19.5000	21.3333
13	9.3077	11.2308	13.1538	15.8462	17.0000	19.8462	22.0769
14	10.2857	10.8571	12.8571	15.2857	16.8571	19.0000	21.4286
15	11.2667	11.2000	13.5333	15.3333	17.4000	19.0667	21.8000

n\k	9	10	11	12	13	14	15
2	-	-	-	-	-	-	-
3	-	-	-	-	-	-	42.0000
4	32.0000	36.0000	40.0000	44.0000	48.0000	52.0000	56.0000
5	40.0000	29.0000	32.4000	35.8000	39.2000	42.6000	34.0000
6	24.0000	27.3333	30.6667	34.0000	37.3333	40.6667	39.0000
7	25.1429	28.7143	32.2857	29.0000	32.0000	35.0000	38.0000
8	25.7500	27.0000	27.7500	28.0000	31.0000	34.0000	37.0000
9	24.0000	27.6667	28.8889	29.6667	32.8889	33.0000	32.6667
10	24.2000	26.0000	27.4000	30.8000	31.6000	34.8000	35.0000
11	24.1818	24.4545	28.0000	29.3636	30.3636	33.5455	36.7273
12	24.0000	26.3333	26.5000	28.0000	31.3333	32.3333	35.5000
13	23.6923	24.6923	28.4615	30.3846	30.0000	33.3077	34.3077
14	23.2857	24.5714	26.8571	28.8571	30.5714	32.0000	33.1429
15	24.0000	25.6667	26.8000	29.0000	30.9333	32.6000	34.0000

TABLE 1 (Continued)

Critical values of χ^2 for $2 \leq k \leq 15$, $2 \leq n \leq 15$, and $p_i = 1/k$

$\alpha = .001$

n\k	2	3	4	5	6	7	8
2	-	-	-	-	-	-	-
3	-	-	-	-	-	-	-
4	-	-	-	-	-	-	-
5	-	-	-	-	25.0000	30.0000	35.0000
6	-	-	18.0000	24.0000	30.0000	36.0000	42.0000
7	-	-	21.0000	28.0000	24.7143	30.0000	26.1429
8	-	16.0000	24.0000	23.2500	22.0000	27.0000	24.0000
9	-	18.0000	19.8889	20.4444	21.0000	26.0000	27.4444
10	-	20.0000	17.2000	19.0000	22.4000	25.0000	25.2000
11	11.0000	16.5455	15.5455	20.3636	21.1818	25.2727	26.0909
12	12.0000	14.0000	18.0000	18.8333	22.0000	24.1667	26.6667
13	13.0000	15.3846	16.8462	19.6923	21.6154	24.1538	25.7692
14	14.0000	13.8571	16.8571	18.8571	21.1429	24.0000	26.0000
15	11.2667	14.8000	16.2000	18.6667	22.2000	22.8000	26.0667

n\k	9	10	11	12	13	14	15
2	-	-	-	-	-	-	-
3	-	-	-	-	-	-	-
4	-	36.0000	40.0000	44.0000	48.0000	52.0000	56.0000
5	40.0000	45.0000	50.0000	55.0000	60.0000	65.0000	70.0000
6	33.0000	37.3333	41.6667	34.0000	37.3333	40.6667	44.0000
7	30.2857	34.4286	38.5714	35.8571	39.4286	43.0000	46.5714
8	28.0000	32.0000	23.2500	37.0000	40.7500	44.5000	44.5000
9	32.0000	29.8889	31.3333	35.0000	38.6667	42.3333	42.6667
10	29.6000	32.0000	34.0000	35.6000	36.8000	37.6000	41.0000
11	27.4545	29.9091	34.0000	35.9091	37.4545	38.6364	42.1818
12	30.0000	29.6667	32.0000	36.0000	37.8333	41.6667	40.5000
13	29.2308	30.8462	31.8461	34.0769	36.0000	39.7692	41.2308
14	28.4286	30.2857	33.1429	35.7143	36.1429	38.0000	39.5714
15	27.6000	29.6667	32.6667	35.4000	37.8667	38.2000	40.0000

TABLE 2

Exact Significance Levels of the Corresponding Critical Values in Table 1

$\alpha = .050$

n\k	2	3	4	5	6	7	8
2	-	-	-	-	-	-	-
3	-	-	-	.0400	.0278	.0204	.0156
4	-	.0370	.0156	.0080	.0046	.0029	.0020
5	-	.0123	.0039	.0336	.0201	.0379	.0259
6	.0313	.0041	.0186	.0272	.0136	.0379	.0259
7	.0156	.0206	.0208	.0323	.0158	.0354	.0219
8	.0078	.0334	.0336	.0366	.0483	.0341	.0198
9	.0391	.0248	.0457	.0321	.0484	.0399	.0403
10	.0215	.0224	.0371	.0398	.0403	.0364	.0345
11	.0117	.0376	.0448	.0316	.0407	.0311	.0275
12	.0386	.0485	.0483	.0395	.0329	.0491	.0421
13	.0225	.0378	.0379	.0454	.0386	.0382	.0435
14	.0129	.0331	.0459	.0336	.0385	.0433	.0453
15	.0352	.0429	.0376	.0467	.0418	.0366	.0493

n\k	9	10	11	12	13	14	15
2	-	-	-	-	-	-	-
3	.0123	.0100	.0083	.0069	.0059	.0051	.0044
4	.0453	.0370	.0308	.0451	.0387	.0335	.0293
5	.0184	.0136	.0103	.0080	.0063	.0457	.0401
6	.0184	.0136	.0438	.0345	.0277	.0225	.0185
7	.0437	.0399	.0299	.0229	.0180	.0143	.0116
8	.0450	.0311	.0221	.0394	.0489	.0388	.0313
9	.0400	.0261	.0176	.0413	.0312	.0484	.0395
10	.0365	.0395	.0490	.0346	.0250	.0184	.0414
11	.0469	.0303	.0368	.0423	.0305	.0451	.0342
12	.0403	.0431	.0438	.0300	.0392	.0479	.0360
13	.0335	.0323	.0355	.0392	.0445	.0320	.0432
14	.0378	.0456	.0448	.0284	.0338	.0385	.0475
15	.0359	.0449	.0381	.0376	.0405	.0477	.0338

TABLE 3

Exact Significance Levels of the Smallest Observable Values of χ^2 which Yield Significance at the .050 Level Using the Continuous Approximation

n\k	2	3	4	5	6	7	8
2	-	-	-	-	-	-	-
3	-	.1111	.0625	.0400	.0278	.0204	.0156
4	.1250	.0370	.0156	.0080	.0046	.0729	.0566
5	.0625	.0123	.0625	.0336	.0201	.0379	.0259
6	.0313	.0535	.0186	.0272	.0586	.0379	.0259
7	.0156	.0206	.0515	.0323	.0158	.0354	.0611
8	.0703	.0590	.0336	.0538	.0483	.0341	.0198
9	.0391	.0504	.0457	.0321	.0484	.0399	.0403
10	.0215	.0590	.0371	.0398	.0403	.0364	.0345
11	.0654	.0376	.0448	.0316	.0407	.0311	.0528
12	.0386	.0704	.0483	.0395	.0329	.0395	.0421
13	.0225	.0378	.0379	.0454	.0386	.0382	.0435
14	.0574	.0582	.0459	.0519	.0385	.0433	.0453
15	.0352	.0429	.0376	.0414	.0418	.0366	.0493

n\k	9	10	11	12	13	14	15
2	.1111	.1000	.0909	.0833	.0769	.0714	.0667
3	.0123	.0100	.0083	.0069	.0059	.0051	.0044
4	.0453	.0370	.0308	.0451	.0387	.0335	.0293
5	.0184	.0856	.0718	.0611	.0526	.0457	.0401
6	.0184	.0568	.0438	.0345	.0277	.0225	.0797
7	.0437	.0399	.0299	.0229	.0180	.0622	.0519
8	.0450	.0663	.0506	.0394	.0489	.0388	.0313
9	.0400	.0261	.0557	.0413	.0602	.0484	.0395
10	.0365	.0395	.0490	.0346	.0673	.0524	.0414
11	.0469	.0548	.0368	.0423	.0305	.0451	.0342
12	.0403	.0431	.0438	.0550	.0392	.0479	.0360
13	.0335	.0571	.0582	.0392	.0445	.0579	.0432
14	.0514	.0456	.0448	.0504	.0542	.0385	.0475
15	.0539	.0449	.0381	.0376	.0405	.0477	.0540

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A table of exact critical values of the chi square criterion of goodness of fit for n observations in k equiprobable categories is presented ($2 \leq n \leq 15$, $2 \leq k \leq 15$). The nominal significance levels are: $\alpha \leq .200$, .100, .050, .025, .010, .005, .001. Exact significance levels are presented for $\alpha \leq .050$, and it is shown that the true significance levels of the standard (continuous) chi square test with $\alpha = .050$ exceed .050 in 27 percent of the cases examined. Thus, while the exact test is uniformly conservative with respect to Type 1 errors, the standard test is not. (U)

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